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Coupling constants $g_{a_0\omega\gamma}$ and $g_{a_0\rho\gamma}$ as derived from QCD sum rules

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Abstract. We consider the two point correlation function of a scalar current in the QCD sum rule approach to estimate the overlap amplitude of the a_0-meson. We then employ QCD sum rules to calculate the coupling constants $g_{\omega a_0\gamma}$ and $g_{\rho a_0\gamma}$ by studying the three point $a_0\omega\gamma$- and $a_0\rho\gamma$-correlation functions.

1 Introduction

The low-mass scalar mesons have fundamental importance in understanding the theory and phenomenology of low energy QCD. From the experimental point of view, the isoscalar $f_0(980)$ and isovector $a_0(980)$ are well established, but the nature and the quark substructure of these scalar mesons, the question whether they are conventional $q\overline{q}$ states [1], $K\overline{K}$ molecules [2], or multiquark exotic $q^2\overline{q}^2$ states [3] has been a subject of controversy. On the other hand, since they are relevant hadronic degrees of freedom, besides the questions of their nature, the roles of scalar mesons in the hadronic processes must be studied.

The radiative decay processes of the type $V^0 \to P^0P^0\gamma$ where V and P belong to the lowest multiplets of the vector (V) and pseudoscalar (P) mesons have become a subject of renewed interest, because they offer the possibility of investigating the new physics features governing meson physics in the low energy region. Although these rare decays have small branching ratios due to the absence of bremsstrahlung radiation, their study offers an opportunity to test the theoretical ideas about the interesting mechanisms of these decays, as well as to shed light on the structure of intermediate states involved in these decays. Particularly interesting are the exchange mechanisms of scalar resonances contributing to these decays. The radiative decays $\rho^0 \to \pi^0\eta\gamma$ and $\omega \to \pi^0\eta\gamma$ were studied using a low energy effective Lagrangian approach with gauged Wess–Zumino terms [4], and later by using standard Lagrangians obeying a $SU(3)$ symmetry [5]. In both of these calculations, scalar meson intermediate state contributions were neglected and the contributions of intermediate vector mesons were taken into account. However, it is of interest to study the contribution of the a_0-intermediate state to these decays as well, and for that knowledge of the $a_0\omega\gamma$- and $a_0\rho^0\gamma$-vertices is needed.

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In this work, we estimate the coupling constants $g_{a_0\rho\gamma}$ and $g_{a_0\omega\gamma}$ by employing QCD sum rules, which provide an efficient method to study hadronic properties and which have been employed to study hadronic observables such as decay constants and form factors in terms of non-perturbative contributions proportional to the quark and gluon condensates [6–8].

2 Analysis and results

The QCD sum rules approach [6–8] is a model independent method to study the properties of hadrons through correlation functions of appropriately chosen currents. We choose the interpolating currents for the ω- and ρ-mesons as $j^\omega_\mu = (1/2)(\overline{u}\gamma_\mu u + \overline{d}\gamma_\mu d)$ and $j^\rho_\mu = (1/2)(\overline{u}\gamma_\mu u - \overline{d}\gamma_\mu d)$ respectively, and for the a_0-meson as $j_{a_0} = (1/2)(\overline{u}u - \overline{d}d)$ [6,7], and we work in the $SU(2)$ flavor limit $m_u = m_d = m_q$. In the sum rule, the overlap amplitude of the a_0-meson, $\lambda_{a_0} = \langle 0|j_{a_0}|a_0\rangle$, is needed. In a previous work [9] we studied the scalar–isoscalar σ-meson by considering the two point scalar current correlation function. Since perturbative and QCD-vacuum condensate contributions to the scalar current correlation functions cannot distinguish between isoscalar and isovector channels, we follow here the same method and we study the scalar–isovector a_0-meson by considering the two point current correlation function

$$\Pi(p^2) = \mathrm{i}\int \mathrm{d}^4x\mathrm{e}^{\mathrm{i}p.x}\langle 0|T\{j_{a_0}(x)j^\dagger_{a_0}(0)\}|0\rangle. \qquad (1)$$

The two-loop expression for the scalar current correlation function $\Pi(p^2)$ in perturbative QCD was calculated [10], and for light quark systems in the limit $m_q = 0$ it is given by the expression

$$\Pi_{\mathrm{pert}}(p^2) = \frac{3}{16\pi^2}(-p^2)\ln\left(\frac{-p^2}{\mu^2}\right) \times\left\{1 + \frac{\alpha_{\mathrm{s}}}{\pi}\left[\frac{17}{3} - \ln\left(\frac{-p^2}{\mu^2}\right)\right]\right\}. \qquad (2)$$

QCD-vacuum condensate contributions to the scalar current correlation function $\Pi(p^2)$ were obtained by the operator product method [11] in the same limit, $m_q = 0$:

$$\Pi(p^2 = -Q^2)_{\rm cond} = \frac{3}{2Q^2}\langle m_q \overline{q}q\rangle + \frac{1}{16\pi Q^2}\langle \alpha_{\rm s} G^2\rangle - \frac{88\pi}{27Q^4}\langle \alpha_{\rm s}(\overline{q}q)^2\rangle. \qquad (3)$$

Let us note that the term $\langle m_q \overline{q}q\rangle$ is independent of the quark mass since it is given as $-f_\pi^2 m_\pi^2/4$ through the Gell-Mann–Oakes–Renner relation [6].

The correlation function $\Pi(p^2)$ satisfies the standard subtracted dispersion relation [6]

$$\Pi_{\rm pert}(p^2) = p^2 \int_0^\infty \frac{{\rm d}s}{s(s-p^2)}\rho(s) + \Pi(0), \qquad (4)$$

where the spectral density function is given by $\rho(s) = (1/\pi){\rm Im}\Pi(s)$. The spectral density contains a single sharp pole $\pi\lambda_{a_0}\delta(s - m_{a_0}^2)$ corresponding to the coupling of the a_0-meson to the scalar current. The continuum contribution of the higher states to the spectral density is estimated to be $\rho = \rho_h(s)\theta(s - s_0)$ where s_0 denotes the continuum threshold and ρ_h is given by the expression $\rho_h(s) = (1/\pi){\rm Im}\Pi_{\rm OPE}(s)$ where $\Pi_{\rm OPE}(s)$ is obtained from (2) and (3): $\Pi_{\rm OPE}(s) = \Pi_{\rm pert}(s) + \Pi_{\rm cond}(s)$. After performing the Borel transformation we obtain the QCD sum rule for the overlap amplitude λ_{a_0}; it is

$$\lambda_{a_0}^2 {\rm e}^{-m_{a_0}^2/M^2} = \frac{3}{16\pi^2}M^4 \left\{ \left[1 - \left(1 + \frac{s_0}{M^2}\right){\rm e}^{-s_0/M^2}\right] \times \left(1 + \frac{\alpha_{\rm s}(M)}{\pi}\frac{17}{3}\right) - 2\frac{\alpha_{\rm s}(M)}{\pi}\int_0^{s_0/M^2} w \ln w {\rm e}^{-w}{\rm d}w \right\} + \frac{3}{2}\langle m_q\overline{q}q\rangle + \frac{1}{16\pi}\langle \alpha_{\rm s}G^2\rangle - \frac{88\pi}{27M^2}\langle \alpha_{\rm s}(\overline{q}q)^2\rangle. \qquad (5)$$

In the numerical evaluation of (5) we use $\langle m_q\overline{q}q\rangle = (-0.82\pm0.10)\times10^{-4}\,{\rm GeV}^4$, $\langle\alpha_{\rm s}G^2\rangle = (0.038\pm0.011)\,{\rm GeV}^4$, $\langle\alpha_{\rm s}(\overline{q}q)^2\rangle = -0.18\times10^{-3}\,{\rm GeV}^6$ [8,12]. The threshold is chosen below a possible $a_0(1450)$ pole and it is varied between $s_0 = 1.6$–$1.7\,{\rm GeV}^2$. Since the Borel parameter has no physical meaning, we look for a range of its values where the sum rule is almost independent of M^2; we choose the interval of values of the Borel parameter M^2 as 1.2–2.0 $\rm GeV^2$. The overlap amplitude λ_{a_0} as a function of M^2 in this interval for different values of s_0 is shown in Fig. 1 from which by choosing the middle value $M^2 = 1.6\,{\rm GeV}^2$ in the interval of variation, we obtain the overlap amplitude as $\lambda_{a_0} = 0.21 \pm 0.05\,{\rm GeV}^2$ where we include the uncertainty due to the variation of the continuum threshold and the Borel parameter M^2 as well as the uncertainty due to errors attached to the estimated values of condensates as quoted above.

In order to derive the QCD sum rule for the coupling constants $g_{a_0\omega\gamma}$ and $g_{a_0\rho\gamma}$, we consider the three point correlation function

$$T_{\mu\nu}(p,p') = \int {\rm d}^4x{\rm d}^4y {\rm e}^{{\rm i}p'\cdot y}{\rm e}^{-{\rm i}p.x} \times \langle 0|T\{j_\mu^\gamma(0)j_\nu^V(x)j_{a_0}(y)\}|0\rangle, \qquad (6)$$

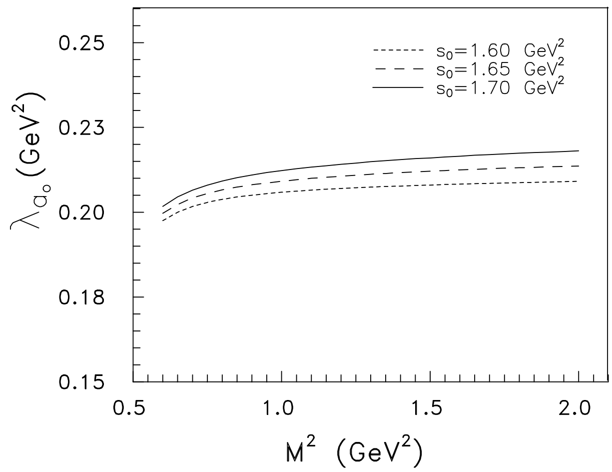

Fig. 1. The overlap amplitude λ_{a_0} as a function of the Borel parameter M^2

where $j_\mu^\gamma = (e_u\overline{u}\gamma_\mu u + e_d\overline{d}\gamma_\mu d)$ is the electromagnetic current with e_u and e_d being the quark charges, and j_ν^V is the interpolating current for the ω- or ρ^0-meson.

In order to obtain the phenomenological part of the sum rule, we consider the double dispersion relation for the vertex function $T_{\mu\nu}$,

$$T_{\mu\nu}(p,p') = \frac{1}{\pi^2}\int {\rm d}s_1 \int {\rm d}s_2 \frac{\rho_{\mu\nu}(s_1,s_2)}{(p^2-s_1)(p'^2-s_2)}, \qquad (7)$$

where the possible subtraction terms are neglected since they will not make any contribution after a double Borel transform. For low values of s_1 and s_2, the spectral function $\rho_{\mu\nu}(s_1,s_2)$ contains a term proportional to the double δ-function $\delta(s_1 - m_V^2)\delta(s_2 - m_{a_0}^2)$, corresponding to the transition $a_0 \to V\gamma$ where V denotes the ω- or ρ^0-meson. We therefore saturate the dispersion relation satisfied by the vertex function $T_{\mu\nu}$ by these lowest lying meson states in the vector and the scalar channels, and in this way we obtain for the physical part

$$T_{\mu\nu}(p,p') = \frac{\langle 0|j_\nu^V|V\rangle\langle V(p)|j_\mu^\gamma|a_0(p')\rangle\langle a_0|j_{a_0}|0\rangle}{(p^2-m_V^2)(p'^2-m_{a_0}^2)} + ..., \qquad (8)$$

where the contributions from the higher states and the continuum is denoted by dots. In this expression the overlap amplitude $\lambda_{a_0} = \langle a_0|j_{a_0}|0\rangle$ of the a_0-meson has been determined in previous sections. The overlap amplitude λ_V of the vector meson is defined by $\langle 0|j_\nu^V|V\rangle = \lambda_V u_\nu$, where u_ν is the polarization vector of the vector meson ω or ρ^0. The matrix element of the electromagnetic current is given by

$$\langle V(p)|j_\mu^\gamma|a_0(p')\rangle = -{\rm i}\frac{e}{m_V}g_{a_0V\gamma}K(q^2)(p\cdot qu_\mu - u\cdot qp_\mu), \qquad (9)$$

where $q = p - p'$, and $K(q^2)$ is a form factor with $K(0) = 1$. This expression defines the coupling constant through the effective Lagrangian

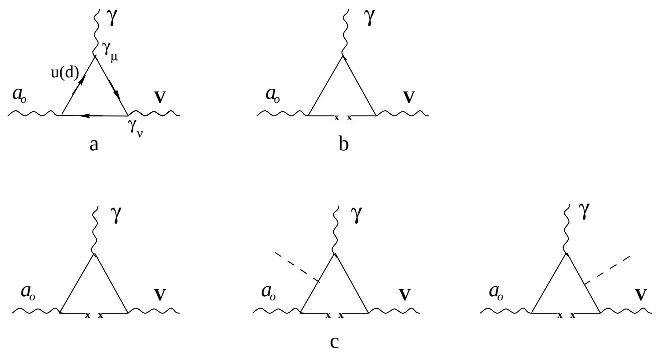

Fig. 2a–c. Feynman diagrams for the $a_0V\gamma$-vertex: **a** bare-loop diagram, **b** $d=3$ operator corrections, **c** $d=5$ operator corrections. The dotted lines denote gluons

$$\mathcal{L} = \frac{e}{m_V} g_{a_0V\gamma}\partial^\alpha V^\beta(\partial_\alpha A_\beta - \partial_\beta A_\alpha)a_0, \qquad (10)$$

describing the $a_0V\gamma$-vertex.

The theoretical part of the sum rule is obtained by calculating the perturbative contribution and the power corrections from operators of different dimensions to the three point correlation function $T_{\mu\nu}$. For the perturbative contribution we consider the lowest order bare-loop diagrams shown in Fig. 2a. Furthermore, we consider the power corrections from the operators of different dimensions that are proportional to the vacuum condensates $\langle\overline{q}q\rangle$, $\langle\overline{q}\sigma\cdot Gq\rangle$ and $\langle(\overline{q}q)^2\rangle$. We do not consider the gluon condensate contribution proportional to $\langle G^2\rangle$, since it is estimated to be negligible for light quark systems. We perform the calculations of the power corrections in the fixed point gauge [13]. We work in the limit $m_q=0$, and in this limit the perturbative bare-loop diagram does not make any contribution. Moreover, in this limit only operators of dimensions $d=3$ and $d=5$ make contributions that are proportional to $\langle\overline{q}q\rangle$ and $\langle\overline{q}\sigma\cdot Gq\rangle$, respectively. The relevant Feynman diagrams for power corrections are shown in Figs. 2b,c. If we consider the gauge invariant structure $(p_\mu q_\nu - p\cdot q g_{\mu\nu})$, we obtain the power corrections of dimensions $d=3$ and $d=5$:

$$C_3 = \mathrm{i}\frac{3}{4}\frac{1}{p^2}\frac{1}{p'^2}(e_u\langle\overline{u}u\rangle + e_d\langle\overline{d}d\rangle) \qquad (11)$$

and

$$C_5 = \left(\mathrm{i}\frac{9}{32}\frac{1}{p^4}\frac{1}{p'^2} + \mathrm{i}\frac{1}{32}\frac{1}{p^2}\frac{1}{p'^4}\right) \times (e_u\langle g_s\overline{u}\sigma\cdot Gu\rangle + e_d\langle g_s\overline{d}\sigma\cdot Gd\rangle). \qquad (12)$$

After performing a double Borel transform with respect to the variables $Q^2=-p^2$ and $Q'^2=-p'^2$, and by considering the gauge invariant structure $(p_\mu q_\nu - p\cdot q g_{\mu\nu})$ for the phenomenological part as well, we obtain the sum rule for the coupling constant $g_{a_0V\gamma}$:

$$g_{a_0V\gamma} = -e_q\langle\overline{u}u\rangle\frac{3m_V}{\lambda_{a_0}\lambda_V}\mathrm{e}^{m_{a_0}^2/M^2}\mathrm{e}^{m_V^2/M'^2} \times\left(\frac{3}{4} - \frac{9}{32}m_0^2\frac{1}{M^2} - \frac{1}{32}m_0^2\frac{1}{M'^2}\right), \qquad (13)$$

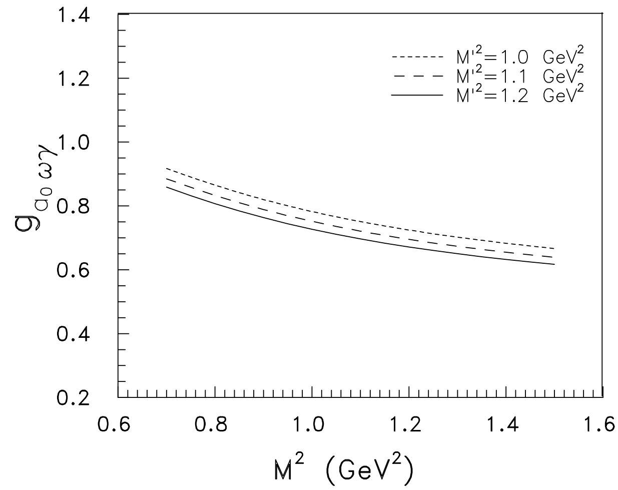

Fig. 3. The coupling constant $g_{a_0\omega\gamma}$ as a function of the Borel parameter M^2 for different values of M'^2

where $e_q=(e_u+e_d)$ for the ρ^0-meson and $e_q=(e_u-e_d)$ for the ω-meson, and we use the relations $\langle\overline{q}\sigma\cdot Gq\rangle = m_0^2\langle\overline{q}q\rangle$ and $\langle\overline{u}u\rangle = \langle\overline{d}d\rangle$. For the numerical evaluation of the sum rule we use the values $m_0^2=(0.8\pm0.02)\,\mathrm{GeV}^2$, $\langle\overline{u}u\rangle = (-0.014\pm0.002)\,\mathrm{GeV}^3$ [8,14], and $m_\rho=0.770\,\mathrm{GeV}$, $m_\omega=0.782\,\mathrm{GeV}$. For the overlap amplitude λ_{a_0} we use the value $\lambda_{a_0}=(0.21\pm0.05)\,\mathrm{GeV}^2$ that we have estimated previously. We determine the overlap amplitude λ_V for the ω- and ρ^0-meson from the measured leptonic decay widths $\Gamma(V\to e^+e^-)$ [15]; thus we use their experimental values $\lambda_\rho=(0.117\pm0.003)\,\mathrm{GeV}^2$ and $\lambda_\omega=(0.108\pm0.002)\,\mathrm{GeV}^2$. In order to analyze the dependence of $g_{a_0V\gamma}$ on the Borel parameters M^2 and M'^2, we study the independent variations of M^2 and M'^2 in the interval $0.6\,\mathrm{GeV}^2\le M^2$, $M'^2\le 1.4\,\mathrm{GeV}^2$ as these limits determine the allowed interval for the vector channel [16]. The variation of the coupling constant $g_{\omega a_0\gamma}$ as a function of the Borel parameter M^2 for different values of M'^2 is shown in Fig. 3, examination of which indicates that it is quite stable with these reasonable variations of M^2 and M'^2. We choose the middle value $M^2=1\,\mathrm{GeV}^2$ for the Borel parameter in its interval of variation and we obtain the coupling constant $g_{a_0\omega\gamma}=(0.75\pm0.20)$. We indicate the error arising from the numerical analysis of the sum rule as well as from the uncertainties in the estimated values of the vacuum condensates. In Fig. 4 we present the variation of the coupling constant $g_{a_0\rho\gamma}$ as a function of the Borel parameter M^2 for different values of M'^2. Following a similar analysis as in the case of $g_{a_0\omega\gamma}$, we obtain the coupling constant $g_{a_0\rho\gamma}=(2.00\pm0.50)$. The values for the coupling constants $g_{a_0\omega\gamma}$ and $g_{a_0\rho\gamma}$ that we obtain are in agreement with the expected $SU(3)$ ratio $g_{a_0\rho\gamma}:g_{a_0\omega\gamma}=3:1$.

In our analysis, we use for the overlap amplitudes λ_ω and λ_ρ the values that we obtain from the experimental electronic decay widths of the ω- and ρ^0-mesons. On the other hand, it may be argued that in a QCD sum rule calculation it is more appropriate to use the values of the overlap amplitudes that are also determined within

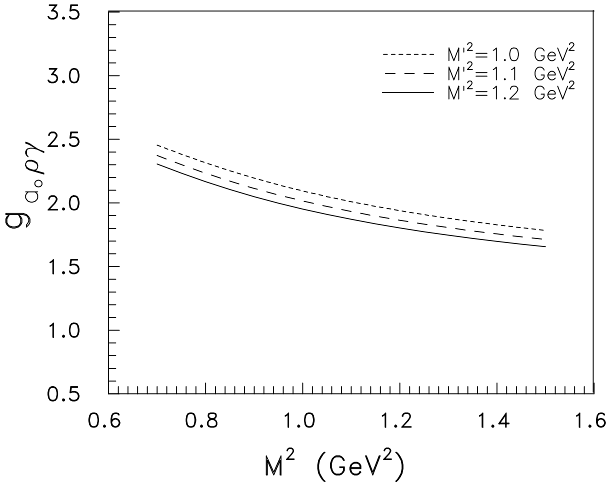

Fig. 4. The coupling constant $g_{a_0\rho\gamma}$ as a function of the Borel parameter M^2 for different values of M'^2

the framework of the QCD sum rule method. Electromagnetic decays of vector mesons using QCD sum rules were studied in [17], and in this analysis the authors used the values of the overlap amplitudes $\lambda_\omega = (0.16 \pm 0.01)\,\mathrm{GeV}^2$ and $\lambda_\rho = (0.17 \pm 0.01)\,\mathrm{GeV}^2$ that they also determined utilizing the QCD sum rules. If we use instead these values of the overlap amplitudes in our calculation, we obtain the coupling constants $g_{a_0\omega\gamma} = 0.45 \pm 0.10$ and $g_{a_0\rho\gamma} = 1.30 \pm 0.30$, which are consistent with our above results.

In the investigations of the role of the a_0-meson in hadronic processes, the relevant coupling constants of the a_0-meson are needed. In this work, we employed the QCD sum rule approach to estimate the coupling constants $g_{a_0\omega\gamma}$ and $g_{a_0\rho\gamma}$. We feel that the studies of the different coupling constants of the a_0-meson should be continued. In particular QCD sum rule calculations should be improved by taking into account the high order corrections to the perturbative part of the three point correlation function and also to the two point correlation function employed in the estimation of the overlap amplitude of the a_0-meson.

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